p-adic Hodge theory homework: Week 4

1. (a) Choose $\alpha \in \mathbb{F}_{p^n}^\times$. Let $\chi\colon \mathrm{Gal}(\overline{\mathbb{F}}_p/\mathbb{F}_{p^m}) \to \mathbb{F}_{p^n}^\times$ be the map sending the p^m-Frobenius to α. Give an explicit description of the étale φ-module associated with χ.

   (b) Choose $\beta \in \mathbb{F}_{p^n}$. Let $\psi\colon \mathrm{Gal}(\overline{\mathbb{F}}_p/\mathbb{F}_{p^m}) \to \mathrm{GL}_2(\mathbb{F}_{p^n})$ be the map sending the p^m-Frobenius to $\begin{pmatrix} 1 & \beta \\ 0 & 1 \end{pmatrix}$. Give an explicit description of the étale φ-module associated with ψ.

2. [BC] exercise 3.4.1.

3. [BC] exercise 3.4.3.