p-adic Hodge theory homework: Week 14

1. [D] problem 35. (Note that the definition of the pro-étale site in [D] is the "old" one from Scholze's p-adic Hodge theory paper. You can use the quasi-pro-étale site instead.)

2. [D] problem 36.

3. Let n, i be nonnegative integers.

   (a) Verify that there is an isomorphism of $\mathbb{Z}_p^\times = \mathrm{Gal}(\mathbb{Q}_p(\mu_{p^\infty})/\mathbb{Q}_p)$-modules
   $$H^i_{\mathrm{cts}}(\mathbb{Z}_p(1)^n, \mathbb{Z}_p) \cong \mathbb{Z}_p(-i)^{\binom{n}{i}}.$$

   (b) Veryify that
   $$H^0\left(\mathbb{Z}_p(1)^n, C\left\langle T_1^{\pm 1/p^\infty}, \dots, T_n^{\pm 1/p^\infty}\right\rangle\right) \cong C\left\langle T_1^{\pm 1}, \dots, T_n^{\pm 1}\right\rangle$$

   (Hint: the argument is similar to the argument of the Ax–Sen–Tate theorem.)

   (c) Verify that the cup product map
   $$H^i_{\mathrm{cts}}\left(\mathbb{Z}_p(1)^n, \mathbb{Z}_p\right) \otimes_{\mathbb{Z}_p} C\left\langle T_1^{\pm 1}, \dots, T_n^{\pm 1}\right\rangle \to H^i_{\mathrm{cts}}\left(\mathbb{Z}_p(1)^n, C\left\langle T_1^{\pm 1/p^\infty}, \dots, T_n^{\pm 1/p^\infty}\right\rangle\right)$$

   is an isomorphism.