p-adic Hodge theory homework: Week 1

1. Read sections 1.1-1.2 of `https://swc-math.github.io/aws/2007/KedlayaNotes11Mar.pdf`, and do Exercise 1.2.6. I also recommend reading sections 1.3-1.7.

2. Let K be a field. Show that $\mathrm{Aut}(K(t)/K) = \mathrm{PGL}_2(K)$, with the discrete topology.

3. Give an example of a finite index subgroup of a profinite group that is not open. (Hint: take the group to be an infinite-dimensional $\mathbb{F}_p$-vector space.)